Multi-objective topology optimisation for acoustic porous materials using gradient-based, gradient-free and hybrid strategies

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When designing passive sound-attenuation structures, one of the challenging problems that arise is optimally distributing acoustic porous materials within a design region so as to maximise sound absorption while minimising material usage. To identify efficient optimisation strategies for this multi-objective problem, we compare several gradient, non-gradient and hybrid strategies. For gradient approaches, the solid-isotropic-material-with-penalisation method (SIMP) and a novel gradient-based constructive heuristic (CHg) are considered. For gradient-free approaches, hill climbing with a weighted-sum scalarisation (HC) and a non-dominated sorting genetic algorithm II (NSGA-II) are considered. Optimisation trials are conducted on seven benchmark problems involving rectangular design domains in impedance tubes subject to normal-incidence sound loads. The results indicate that while gradient methods can provide quick convergence with high-quality solutions, often gradient-free strategies are able to find improvements in specific regions of the Pareto front. Two novel hybrid approaches (HA1 and HA2) are proposed combining a gradient method (CHg) for initiation and a non-gradient method (respectively HC and NSGA-II) for local improvements. A novel and effective Pareto-slope-based weighted-sum hill climbing is introduced for local improvement. Results reveal that for a given computational budget, the hybrid methods can consistently outperform the parent gradient or non-gradient methods.

Keywords: Acoustic porous materials; topology optimisation; sound absorption; multi-objective optimisation;

I. INTRODUCTION

Acoustic porous materials such as foams or fibrous materials are widely used for passive noise control in automotive, aerospace and construction industries. While these materials generally exhibit sound absorption across wide frequency bands, their low-frequency absorption performance is poor since the lengths of the absorber typically needed are higher for longer wavelengths[1]. To alleviate this problem, one can modify the absorber shape or introduce macro-scale air cavities[2] to alter the dynamic properties creating favourable resonances that improve absorption while also reducing the material usage. However, optimising the size, shape and placement of these air cavities or other solid scattering materials[3] is essentially a topology optimisation problem[4], which is challenging to solve.

Topology optimisation is the concept of simultaneously optimising both the topology (number of holes in a structure) and the shape (geometry and dimensions of these holes) of mechanical structures so as to maximise the load-bearing capacity with minimal material usage. It is a concept first introduced by Bendsøe and Kikuchi[5,6] in the 1990s and has remarkable potential benefits in terms of reduced weight and costs. In the last two decades, topology optimisation techniques have been extended to automatic generation of optimised acoustic shape designs in various applications, such as horns[7], room sound treatments[8], anechoic chamber foams[2,9], mufflers[10–13], sound barriers[14–17], and car internal cavities[18] to name a few. Although topology optimisation is inherently a multi-objective problem i.e., simultaneously maximising performance and minimising weight, it has been common to treat it as a

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single-objective problem i.e., maximising the performance while using a constraint on the weight. Given that one of the main benefits is the potential weight savings, it is of interest to treat it as a multi-objective problem and obtain multiple trade-off designs simultaneously. The acoustic designers can then choose from the set of Pareto optimal or trade-off solutions for manufacture.

While new and improved optimisation strategies are being published for particular applications, there is a need for comparison studies which would facilitate engineers to choose effective strategies for their use case. Performing such comparisons is challenging since many optimisation paradigms exist to solve topology optimisation problems that vary in solution representation (discrete or continuous), gradient usage, memory (single point or population-based), move operators, acceptance strategies etc. To ensure a fair comparison, each algorithm needs to be applied in the best or most reasonable settings tuned to the problem. In this article, a few selected approaches that are popular and likely to be used by other researchers are tested and compared. The list of approaches chosen are:

- Solid isotropic material with penalisation (SIMP)
- Constructive heuristic with gradient (CHg)
- Hill climbing with weighted-sum scalarisation (HC)
- Non-dominated sorting genetic algorithm-II (NSGA-II)

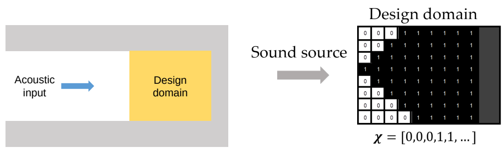
SIMP is the most commonly-used approach for structural topology optimisation[19–21]. A key attribute of this approach is the relaxation of the discrete problem into a continuous problem by allowing intermediate materials and using a power-law interpolation scheme. Using continuous relaxation allows the possibility of computing the gradients quickly using adjoint-like methods, which can make the optimisation quite effective, notwithstanding certain drawbacks such as getting stuck at local optima or the presence of intermediate materials in the final solution. Its effectiveness and ease of implementation[22], have made it the most popular approach for topology optimisation. At this point, it is worth noting some previous efforts toward extending SIMP for multi-objective topology optimisation. Suresh et al.[23] extended the 99-line MATLAB code to a 199-line code for Pareto-optimal compliance minimisation, and also studied the effect of restarts vs hot starts. Hence, in this article, two variants SIMPsweep and SIMPrestart are considered. Mirzendehdel et al.[24] proposed a multi-objective algorithm for multi-material compliance minimisation removing the mass constraint and treating it as an objective. While the multi-objective consideration is prevalent, it constitutes a small fraction of the publications, and comparison studies are rare.

Constructive heuristics are a class of optimisation algorithms that start from empty solutions and build them step by step using specific move operations to reach a complete solution. An example of a constructive heuristic for topology optimisation is the (bi-directional) evolutionary structural optimisation methods (ESO/BESO) introduced by Xie and Steven[25,26]. For compliance minimisation, ESO starts from a completely solid-filled design domain and incrementally removes material from low-stress regions. For acoustic material topology optimisation, Ramamoorthy et al.[9] introduced two constructive heuristics: CH1, where the material is added incrementally to an empty domain in places of highest absorption increase; and CH2, where the material is incrementally removed from a filled domain from places where the decrease in absorption is minimal. These heuristics performed among the top strategies in the study. One of the drawbacks of CH1 and CH2 is that computing the numerical absorption increments is expensive, and this can be overcome by making use of the gradients. Adopting this, a simple gradient-based constructive heuristic (CHg) is proposed in the current study.

Hill climbing is a single objective optimisation technique that starts with an initial solution and modifies it iteratively while accepting improving changes. A row-wise hill climbing approach was found to perform among the best strategies for acoustic material absorption maximisation[9]. A common strategy to solve multi-objective problems is to combine the objectives into a scalar value in a process known as scalarisation[27], and to apply a single objective algorithm. A simple way to scalarise is to use the weighted sum of the objectives. By varying the weights, the relative importance of each objective can be controlled. In this study, hill climbing is used in conjunction with a weighted-sum scalarisation technique (HC) as a candidate for multi-objective topology optimisation.

The non-dominated sorting genetic algorithm-II (NSGA-II) introduced by Deb et al.[28] is a well-known multi-objective evolutionary algorithm. A notable attribute of NSGA-II is the use of a fast non-dominated sorting procedure in combination with a crowding-distance operator that allows finding multiple points in the Pareto front simultaneously, as opposed to having to run multiple trials of a single objective algorithm in combination with a scalarisation technique. The effectiveness of NSGA-II and its variants has made it the most popular multi-objective approach for solving combinatorial optimisation problems[29].

In addition to the above strategies, two hybrid approaches (HA1 and HA2) are proposed involving a gradient method for initialisation and a non-gradient method for local improvement. The aim



(a) (b)

Figure 1. (color online) (a) Schematic of an acoustic system with the design domain. (b) Binary representation of a sample shape. 0 refers to air and 1 refers to porous material.

is to find whether hybrid approaches are beneficial. The results will provide perspectives on each method, and guide algorithm selection.

The article is organised as follows. In section II, the overall methodology including problem description, optimisation formulation, modelling method, and details of the experimental design is provided. In section III, a comparison of gradient algorithms —SIMPsweep, SIMPrestart and CHg is provided. In section IV, a comparison of gradient-free algorithms HC, and NSGA-II are provided. Along with gradient-free algorithms, a random search procedure is also compared. In section V, two hybrid approaches HA1 and HA2 are described and compared with their parent approaches. Finally, in section VI, a summary of the findings and some general guidelines to design algorithms are provided.

II. METHODOLOGY

A. Problem formulation

Consider the problem of optimally filling a rectangular design domain with a given porous material such that the sound absorption is maximised while using minimal material. The design domain can be assumed to be backed by rigid walls with normal-incidence acoustic source placed as shown in Figure 1(a). Sound absorption is the ratio of energy absorbed to the total input sound energy. If no porous material is placed in the design domain, there would not be any absorption. Typically as more porous material is filled in the design domain, the absorption would increase, but this is not always the case. There are instances when removing material would improve absorption[9]. Depending on the distribution of porous material and air in the design domain, sound absorption will be determined at different frequencies of the acoustic source. Thus, this is a classic bi-objective optimisation problem with trade-off solutions.

While there are many ways to formulate the topology optimisation problem, one of the classical ways is to use a fixed discretisation of the system and optimising the material assigned to each finite element. The shape and topology can be represented by a vector $\boldsymbol{\chi}$ with zeros and ones corresponding to the absence or presence of porous material in each element respectively, as shown in Figure 1(b). This is sometimes referred to as a bit-matrix representation[30]. At this point, it is also worth acknowledging other formulations such as moving morphable components[31], level-set method[32,33] etc. The objective considered is to find the optimal discrete assignments of either air or a given poroelastic material to each finite element that simultaneously maximises the normal sound absorption and minimises the volume fraction of the porous material. Mathematically, this formulation can be written as:

Simultaneously,

$$\max_{\boldsymbol{\chi}} \quad \overline{\alpha}(\boldsymbol{\chi}) = \frac{1}{n_f}\sum_{i=1}^{n_f} \alpha(\boldsymbol{\chi}, f_i) \tag{1}$$

$$\min_{\boldsymbol{\chi}} \quad V_f(\boldsymbol{\chi}) = \frac{1}{n_e}\sum_{i=1}^{n_e} \chi_i \tag{2}$$

$$\boldsymbol{\chi} \in \{0,1\}^{n_e}$$
$$\overline{\alpha} \in [0,1]$$
$$V_f \in [0,1]$$

The first objective $\overline{\alpha} \in [0,1]$ is the sound absorption averaged across the target frequencies $(f_1, f_2, ...f_{n_f})$, and the second objective V_f is the porous volume fraction. Absorption $\overline{\alpha}$ is averaged over a number of target frequencies n_f, and porous material volume fraction V_f is averaged over the number of elements n_e in the design domain.

B. Computing the objectives

Computing the volume fraction V_f for a given shape $\boldsymbol{\chi}$ is quite straightforward from Equation 2, whereas computing absorption $\overline{\alpha}$ is computationally expensive requiring solving a system of linear equations. The procedure followed to compute absorption is the same as outlined in Ramamoorthy et al.[9]. The acoustic system is modelled using the unified Biot-Helmholtz model introduced by Lee et al.[2], which considers air as a poroelastic material with negligible solid-part behaviour. In the unified model, air is considered to have $\chi_{air} = 0.001$ to avoid numerical issues when solving the system. Lee et al. also verified the validity of such modelling for poroelastic materials with mixed formulations[34].

The most expensive part of computing $\overline{\alpha}$ is finding the solution $\{\mathbf{X}\}$ to a system of linear equations $[\tilde{\mathbf{S}}(\boldsymbol{\chi}, f)]\{\mathbf{X}\} = \{\tilde{\mathbf{F}}\}$, where the system matrix $[\tilde{\mathbf{S}}(\boldsymbol{\chi}, f)]$ is a square symmetric complex-valued matrix with dimensions of the order of the number of finite elements in the design domain, and $\{\tilde{\mathbf{F}}\}$ is the dynamic forcing vector of the same dimension. The system matrix $[\tilde{\mathbf{S}}(\boldsymbol{\chi}, f)]$ is populated with material properties of air or porous material at specific submatrices depending on the shape $\boldsymbol{\chi}$. When considering continuous relaxation, for the intermediate materials i.e. $\chi_i \in (0, 1]$, the material properties are interpolated using a power-law i.e. any material property, say ψ_i is given by $\psi_{air} + \chi_i^p(\psi_{por} - \psi_{air})$, where ψ_{por} and ψ_{air} are the properties of the porous material and air respectively.

Since evaluating absorption $\overline{\alpha}$ is the computational bottleneck and other algorithmic processes take a relatively insignificant amount of time, this is an expensive optimisation problem, and hence it is reasonable to use the number of absorption evaluations to benchmark the performance of algorithms.

Computing the gradient of sound absorption with respect to the design variables takes approximately two more instances of solving the system of linear equations, making it twice as expensive as computing absorption:

$$\text{timeToCompute}\left(\frac{\partial\overline{\alpha}}{\partial\boldsymbol{\chi}}\right) = 2 \times \text{timeToCompute}(\overline{\alpha}) \tag{3}$$

Such a quick computation of the gradient is achieved using a fictitious load vector pre-multiplication, as explained in Lee et al.[12]. Thus, computing both absorption and the gradient is 3 times as expensive as computing just absorption. Therefore, the gradient methods will be given one-third the fitness evaluation budget.

C. Benchmark problem instances

To compare the optimisation approaches, seven benchmark problem instances previously introduced in Ramamoorthy et al.[9] are adopted. The only difference here is that a modification has been made in the mesh size in problem instance 3 in order to improve the model accuracy. For completeness, the details of the problem instances are provided in Table I. All the problem instances have a rectangular design domain but with varying discretisation, the porous material filled, frequency range of interest, and dimensions. Table II provides the poroelastic material properties for the materials used in the problem instances. While the problem instance 1 uses the same material as Lee et al.[2] with a high tortuosity, the third problem instance uses a fictitious material with high airflow-resistivity, and all other problem instances use melamine.

Table I. Benchmark problems (see section II C)

Problem instance	Mesh size nelx × nely	Length D (m)	Height d (m)	f_{min} Hz	f_{step} Hz	f_{max} Hz	Material ID (see Tab. II)
1	10 × 10	0.135	0.054	100	100	1500	(1)
2	15 × 10	0.045	0.1	100	100	1500	(2)
3	**50 × 20**	0.1	0.1	50	50	500	(3)
4	10 × 10	0.02	0.1	100	100	1500	(2)
5	10 × 10	0.02	0.1	2000	1000	5000	(2)
6	50 × 20	0.135	0.054	100	100	1500	(2)
7	10 × 5	0.135	0.054	500	500	500	(2)

Table II. Materials used in the benchmark problems and their properties (see Table I).

Material parameters	Material-1	Material-2	Material-3
Material:	LKKK[2]	Melamine	High-resistivity foam
Acoustic model:	JCAL[35–37]	JCAL	JCAL
ϕ	0.9	0.99	0.8
Λ' (μm)	449	196	100
Λ (μm)	225	98	10
σ (N·s·m^{-4})	25000	10000	300000
α_∞	7.8	1.01	3
k_0'	4.75e-09	4.75e-09	4.75e-09
ρ (kg·m^{-3})	31.08	8	80
E (Pa)	800000	160000	30000
ν	0.4	0.44	0.44
η	0.265	0.1	0.01

D. Experimental design

Table III provides a quick summary of the optimisation approaches used in this study along with a short description and pseudocode of each approach. More detailed descriptions of each algorithm are provided in the following sections. Reasonable effort has been made to use each algorithm in its recommended or best settings from parameter tuning and has been used in the standard way unless otherwise stated.

All the strategies were given the same arbitrarily-chosen computational budget of 4096 equivalent gradient-free fitness evaluations. Gradient algorithms are assigned $4096/3 \approx 1365$ fitness evaluations, and the non-gradient methods are allowed 4096 fitness evaluations. For the hybrid algorithms, 25% of the computational effort was allotted for gradient-based search and 75% for non-gradient search i.e., $25\% \times 4096/3$ gradient-included and $75\% \times 4096$ gradient-free fitness evaluations.

It should be noted, that in some trials on some problem instances, the resulting SIMP solutions had intermediate materials. In such scenarios, only the non-dominated solutions were discretised by a round-off filter and the fitnesses were recomputed. This is done so that all solutions compared in this study are from the discrete space to facilitate a fair comparison.

To quantify and compare the non-dominated solution set produced by each algorithm, a hypervolume metric will be used. The hypervolume value corresponding to a given set of trade-off solutions is the scalar value equal to the union of

Table III. Optimisation approaches and their settings

Algorithm	Description and pseudocode	Deterministic or stochastic	Trials	Fitness evaluation budget per trial
Gradient-based approaches				
SIMPrestart	Solid isotropic material with penalisation (SIMP) restarted with different volume fraction constraints fixed for a trial: A gradient-based strategy with optimality criteria move-update; following[38]. Initialised with an empty design domain; Restarted with a new $\bar{V}_f$ until budget is used up.	Stochastic: multiple restarts within trial	1 (multiple restarts)	1365 (with gradient)
SIMPsweep	SIMP with adaptive volume fraction constraint: Initialised with an empty design domain; Volume fraction constraint $\bar{V}_f$ updated after each fitness evaluation reached 1 as budget approaches.	Deterministic	1	1365 (with gradient)
CHg	Gradient-based constructive heuristic: Start from an empty solution; Add porous material in steps of 'm' elements where the gradient is highest, until all elements are porous	Deterministic	1	min(N/m,1365) (with gradient)
Non-gradient approaches				
HC	Hill climbing: Use a weighted-sum scalarisation technique to combine the two objectives into a single fitness value. Apply first improvement hill climbing starting from a random discrete solution. Move order is like in a raster-scan.	Stochastic, since initial solution is random	15	4096 (non-gradient)
NSGA-II	Non-dominated sorting genetic algorithm -II[28]: Use a bit representation, tournament selection based on crowding distance and rank, uniform crossover, bitwise mutation probability of $1/N$.	Stochastic	15	4096 (non-gradient)
RAND	Random search algorithm: Picked a desired volume fraction uniformly $\in [0, 1]$; Use this as the probability of porous material at each element and synthesise a solution. Repeat budget number of times.	Stochastic	15	4096 (non-gradient)
Hybrid approaches				
HA1	Hybrid approach 1: Run CHg using 25% of the budget, and run hill climbing for 75% of the budget starting from a selected solution with scalarisation weight such that the combined objective isoline at the solution point in objective space is tangential to the Pareto front.	Deterministic but depends on the point picked for hill climbing	15	4096 (equivalent non-gradient)
HA2	Hybrid approach 2: Run CHg using 25% of the budget, and run NSGA-II for 75% of the budget starting from an initial population from equispaced points in the CHg Pareto front.	Stochastic	15	4096 (equivalent non-gradient)

volumes in the objective space dominated by each solution over and above the objective values of a given reference solution. An illustration is shown in Figure 2. For the bi-objective problem under study, the hypervolume would simply be the area of the objective space that is dominated by the Pareto set obtained from the algorithms from a reference point. The reference point chosen is $(\overline{\alpha}, V_f) = (0, 1)$. Larger the hypervolume, the better the multiobjective performance can be considered to be.

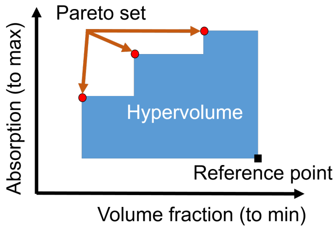


Figure 2. (color online) An illustration of the hypervolume metric.

III. GRADIENT APPROACHES

A. Solid-isotropic-material-with-penalisation(SIMP)

Solid-isotropic-material-with-penalisation (SIMP) is a popular strategy for structural topology optimisation where the main idea is to consider a continuous relaxation of the material choices by using a power-law interpolation scheme. SIMP makes use of gradients to make incremental changes to the shape followed by the application of morphological filters[39]. In this paper, the implementation is adapted from the efficient 88-line code for compliance minimisation by Andreassen et al.[38] replacing compliance and its gradients with absorption and its gradients, and making the material choices as air and porous material instead of solid and void. SIMP takes the desired volume fraction ($\bar{V}_f$) as one of its algorithmic parameters. Two variants are considered namely, SIMPrestart and SIMPsweep.

1. SIMPrestart

In SIMPrestart, multiple trials of SIMP are run with each trial using a different $\bar{V}_f$. For each of these trials, SIMP was initialised from a random solution normalised to have an overall initial volume fraction close to the chosen $\bar{V}_f$. Once convergence is achieved, SIMP is restarted with a new $\bar{V}_f$ and a newly generated initial solution. Depending on $\bar{V}_f$ and the initial solution, the algorithm converges to a variety of shapes as Figure 3(a) shows for problem instance 6. Each trial converged after about 100 iterations. The process is continued until the budget of 1365 is used up.

To populate the Pareto front, equispaced values of $\bar{V}_f$ were used in each trial. The solution progress in the objective space from SIMPrestart for all trials are shown in figure 3(b) for problem instance 6.

2. SIMPsweep

SIMPsweep starts from an empty or air-filled solution with an initial volume fraction limit $\bar{V}_f = 0$, and applies SIMP move updates while updating $\bar{V}_f$ in every iteration reaching $\bar{V}_f = 1$ as the fitness evaluation budget is reached. The solutions produced for problem instance 6 are plotted in the objective space in Figure 4, along with some of the shapes. It can be observed that as the volume fraction increases, the general trend is that absorption also increases. Notably for this melamine problem instance, some of the optimal shapes closely resemble flat layers. Whereas this is not always the case across problem instances.

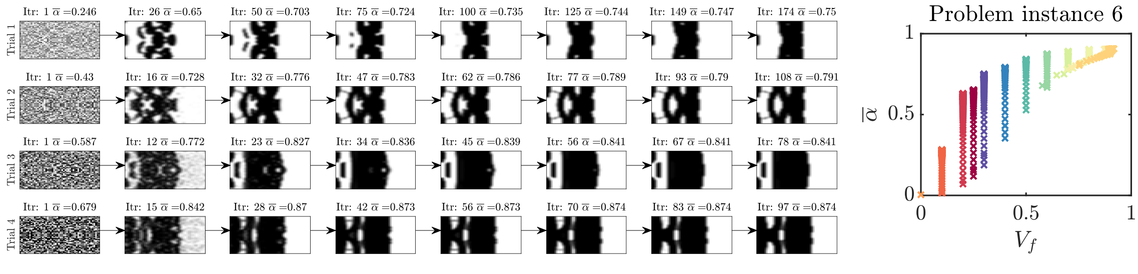
The solutions from SIMP algorithms did not always result in 0 or 1 shapes, and the shapes were rounded i.e., values less than 0.5 are set to 0 and more than 0.5 are set to 1, and the absorptions were recomputed. This involved additional fitness evaluations beyond the budget. Nevertheless, the resulting changes in absorption due to rounding were insignificant in most cases.

A comparison of Pareto fronts of SIMPsweep and SIMPrestart is shown in Figure 6 for problem instance 6. It may be observed that for some volume fraction values ($V_f \approx 0.1$) SIMPsweep found better solutions while in others ($V_f \approx 0.6$) SIMPrestart did. In this problem instance, SIMPsweep seems to cover a larger hypervolume. However, upon observing the hypervolumes for all problem instances in Table V, there seems to be no clear winner between SIMPrestart and SIMPsweep since the former covered more hypervolumes in three problem instances while the latter covered more in the other four.

Among the two, for lower fitness evaluation budgets, SIMPsweep is recommended since unlike in SIMPrestart, less computational time will be spent on initially reaching good solutions as also suggested by Suresh[23].

B. Constructive heuristic using gradient (CHg)

Constructive heuristics are methods which incrementally build solutions from scratch. In a previous study, a material-addition constructive heuristic (CH1) performed among the best approaches in topology optimisation for maximising sound absorption[9]. In CH1, the procedure was to incrementally add porous materials to locations where the increase in absorption would be the highest. However, finding the change in absorption at every finite element is computationally expensive and in this approach (CHg), they are replaced by gradients which are relatively cheap (Equation 3). CHg starts from an empty or air-filled design domain, and fills porous material incrementally in finite elements where the gradient of sound absorption $\frac{\partial \bar{\alpha}}{\partial \chi_i}$ is highest. At each step m number of elements are chosen to fill with porous material after each gradient evaluation, and the total number of fitness evaluations necessary would be n_e/m where n_e is the total number of elements. m is chosen such that n_e/m does not exceed the budget. Note that in the seven problem instances considered, the number of elements are respectively 100, 150, 1000, 100, 100,1000, and 50. Since the bud-



(a) (b)

Figure 3. SIMPrestart:(a) Best shapes from the first 4 trials on problem instance 6 with volume fraction limits 0.3, 0.4, 0.5 and 0.6 respectively. In these shape images and others, the rigid backing is on the right and the acoustic forcing is on the left. (b) Progress in objective space for various trials. Each colour corresponds to a different trial with different $\bar{V}_f$.

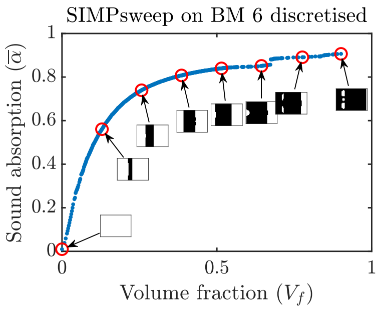


Figure 4. (color online) SIMPsweep: Pareto front for problem instance 6, which uses melamine, results in shapes that resemble flat layers.

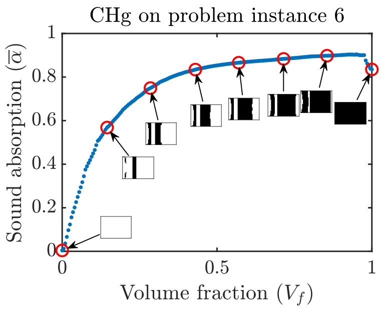


Figure 5. (color online) Solution progress for constructive heuristic using gradients (CHg) applied on problem instance 6

get considered is 1365, all problem instances can be completed in n_e fitness evaluations with $m = 1$. Hence, CHg will effectively utilise less fitness evaluations than the budget in the cases considered. Note that CHg always will search solutions in the discrete space since an element is either filled or not filled. In this way, it is different from SIMPsweep.

The progress of solutions found by CHg applied on problem 6 instance is shown in Figure 5 in the objective space along with a few shapes. Here, the shapes have two flat layers as opposed to one as found in SIMPsweep.

C. Comparing gradient-based approaches

Figure 6 compares the Pareto fronts produced by SIMPrestart, SIMPsweep and CHg algorithms for problem instance 6 as an example. Note that while SIMPrestart tends to leave gaps in the Pareto front, SIMPsweep and CHg finds more solutions and span the front well. There are specific regions where one algorithm performs better than the other two, but overall, these three approaches can be considered to be similar in terms of performance.

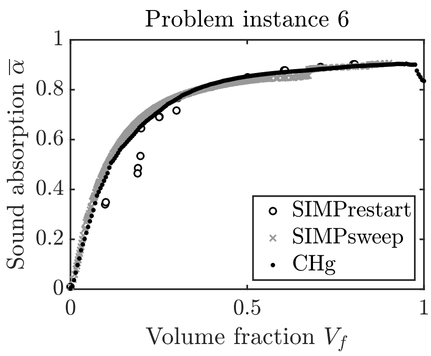


Figure 6. Comparison of gradient methods SIMPrestart, SIMPsweep and CHg for problem instance 6.

The hypervolumes covered by solutions from the gradient approaches are shown in Table IV. Among the three methods, SIMPrestart covered the most hypervolume in one problem instance, SIMPsweep in two problem instances and CHg in the other four, as emphasised by the bold font. However, the values are not significantly different among the three approaches.

Table IV. Hypervolume comparison of gradient based approaches SIMPrestart, SIMPsweep and CHg

Instance	SIMPrestart	SIMPsweep	CHg
1	**0.7065**	0.6835	0.6724
2	0.4014	0.4047	**0.4066**
3	0.7317	0.6063	**0.7412**
4	0.1160	**0.1188**	0.1087
5	0.5208	0.5292	**0.5323**
6	0.7202	**0.7607**	0.7512
7	0.8727	0.8567	**0.8733**

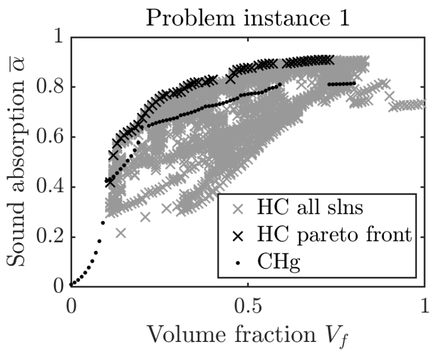


Figure 7. Solutions traversed by hill climbing (HC) with combined Pareto front from 15 trials compared against CHg Pareto front. HC finds improvements over CHg.

An important aspect to note is the possibility to speed up SIMPsweep and CHg if required. For instance, if only $1/10^{th}$ of the fitness evaluation budget is allowed, in SIMPsweep, the volume fraction constraint $\bar{V_f}$ would be adapted 10 times more quickly to reach 1 as the budget is used up. Similarly for CHg, one can simply increase m which is to add more elements with porous material in each iteration. Though this risks potentially missing several trade-off solutions, the quality of the solutions would not be significantly affected. This is because, every next solution found by SIMPsweep or CHg is an incremental perturbation from an already good solution. Although, for SIMPrestart, speed-up can be achieved by tuning the move limit parameter m[38], there are some caveats to doing this such as the occurrence of numerical oscillations.

IV. NON-GRADIENT APPROACHES

A. Hill climbing

Hill climbing is a heuristic for single objective optimisation. Typically, a single initial solution is picked and iteratively modified, and the modified solution is accepted as the current solution if it is improving.

In this implementation, to allow choosing initial solutions spread out in volume fraction, a desired volume fraction is first picked randomly between 0 and 1, and this value is used as the probability to fill porous material in each element.

From the initial solution, elements are bit-flipped row-by-row, and the change is accepted if the scalarised objective function decreases. This is similar to HC in Ramamoorthy et al.[9] but with a weighted-sum scalarisation, in which the two objectives are combined into one as given in Equation 4.

$$\min_{\boldsymbol{x}} \quad C = -w\overline{\alpha} + (1-w)V_f \tag{4}$$

The weight w corresponds to the importance of maximising absorption as opposed to minimising volume fraction and can take values between 0 and 1. A weight of 1 implies maximising only absorption irrespective of volume fraction, and likewise, a weight of 0 corresponds to only minimising volume fraction. An illustration of the effect of choosing w on the scalarised objective is shown in Figure 8. Note that w governs the slope of the isolines of the scalarised objective. This will be relevant later.

For each trial run of HC, a fixed weight is chosen. Then, hill climbing on the combined objective is done until the fitness evaluation budget is used up. 15 such trials are run with different weights. Figure 7 shows all solutions from 15 trials of HC for problem instance 1 compared with CHg solutions. The trails of points in the figure correspond to individual trials improving solutions in a specific direction depending on the chosen weight. The combined results from HC are better than those of CHg in some regions in both $\overline{\alpha}$ and V_f, indicating that the gradient methods do often converge to local-optimal solutions, and potential for improvements exist.

An issue with HC is that only a specific region in the Pareto front will be explored in a given trial. The trial-averaged hypervolumes are significantly lower than the combined hypervolume over 15 trials as may be observed by comparing the HC columns in Tables V and VI. This is because using a set scalarisation weight for a trial guides the search towards a specific region in the Pareto front.

B. Non-dominated sorting genetic algorithms (NSGA-II)

NSGA-II is a popular multi-objective optimisation strategy introduced by Deb et al[28]. It has been effectively used in solving multi-criteria decision-making problems across a plethora of fields. In this implementation, a single-point cross over with an individual cross-over probability of 0.9 is applied with a bit-wise mutation rate of $(1/n_e)$ where n_e is the chromosome length and a popula-

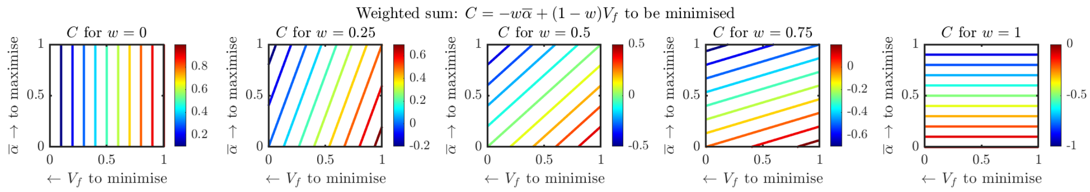


Figure 8. (color online) The effect of weights in weighted-sum scalarisation on the slope of the isolines of combined objective value.

tion size of 32. These parameters were found using parameter-tuning studies on genetic algorithms[9]. Figure 9 shows the progress of solutions in the objective function space for one trial of NSGA-II for problem instance 1. In the figure, each point refers to a particular shape and the colour corresponds to the generation in which it was found. We can observe that as the generations progress (from blue towards red), the solutions tend towards more sound absorption and less volume fraction.

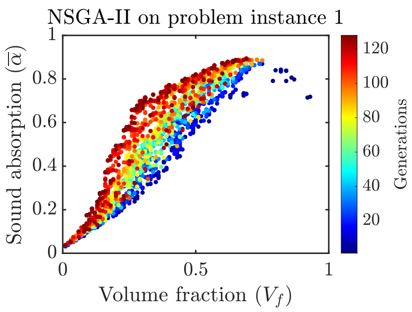


Figure 9. (color online) NSGA-II progress of solutions in the objective function space for problem instance 1 trial 1.

C. Random Search (RAND)

For benchmarking the performance of HC and NSGA-II, a random search algorithm referred here as RAND is applied on all seven problem instances. Random solutions spread across volume fraction are obtained by choosing a random number for desired volume fraction, and using this value as probability to fill porous material in each element. 4096 such solutions are generated and fitnesses are evaluated in each trial, and 15 such trials were run. Using non-dominated sorting on each trial separately and across all 15 trials, the trial-averaged and 15-trial-combined hypervolumes were found and populated in tables V and VI.

D. Comparison of non-gradient algorithms

1. Performance per trial

Comparing the median-trial hypervolumes from HC and NSGAII in Table V, it is clear that NSGA-II is consistently better across all problem instances. This is because based on the choice of scalarisation weight, in a given trial, HC only explores a specific region in the Pareto front. Whereas, NSGA-II spans the objective space effectively due to the crowding distance-based selection mechanism. NSGA-II also outperforms RAND in all problem instances, but interestingly, HC on a per-trial basis, does not outperform even RAND. Moreover, RAND outperforms HC across all problem instances. This is because HC in a single trial is essentially a single-objective algorithm that does not incentivise spanning the hypervolume.

2. Performance across 15 trials combined

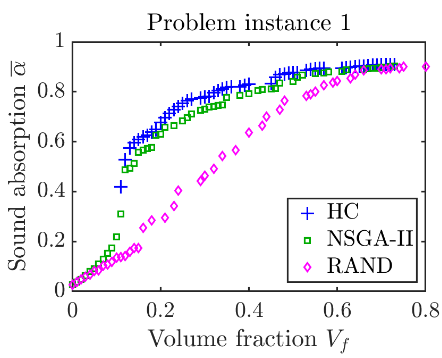


Figure 10. (color online) Combined Pareto fronts from 15 trials of HC, NSGA-II and RAND on problem instance 1.

Combining 15 trials of HC run with different weights results in a better hypervolume than combined results of 15 trials of NSGA-II consistently across all problem instances as can be observed in Table VI (see columns HC vs NSGA-II). As an example, for problem instance 1, by comparing the Pareto fronts in Figure 10, it is clear that HC solutions often have better absorption for the same volume fraction than NSGA-II. Both NSGA-II and HC cover a larger hypervolume than RAND by a large margin.

V. HYBRID APPROACHES

From the studies on gradient and non-gradient algorithms, it was evident that gradient methods can quickly approximate the Pareto front, whereas non-gradient methods can provide improvements in specific regions of the Pareto front.

In order to obtain the benefits of both, two hybrid approaches combining a gradient-based algorithm for initiation and a non-gradient algorithm for improvement is presented and compared. The first hybrid approach is a combination of CHg and HC denoted as HA1, and the second hybrid approach is a combination of CHg and NSGA-II denoted as HA2. We picked CHg as the initiator mainly because, it guarantees discrete solutions and allows the possibility to speed up (see III C).

A. Hybrid approach 1: CHg+HC

Hybrid approach 1 (HA1) combines the use of CHg for 25% of the budget and HC for the remaining 75% of the budget. These numbers are arbitrarily chosen with some basis on experience. Since CHg is gradient-based, and gradient-included evaluations are thrice as expensive as non-gradient fitness evaluations (Equation 3), the rationing is such that CHg uses $25\% \times (\frac{4096}{3})$ fitness evaluations and HC uses $75\% \times (\frac{4096}{1})$.

Figure 11 illustrates the procedure involved in HA1. Firstly, CHg is run to obtain a trade-off solution set. Then, 15 solutions are selected from the CHg trade-off set equispaced in volume fraction to use as initial solutions for each of the 15 HC trials. For each HC trial, a different scalarisation weight w is used such that the isolines of the combined objective C has a slope tangential to CHg Pareto front at the initial solution. The slope of the Pareto front at the initial solution is obtained using a simple central difference of adjacent points. This 'Pareto-slope-based scalarisation' effectively guides HC to find improvements to the Pareto front. HC is run until the remaining budget is used up. As seen in Figure 11, in each trial, only a specific region is explored. The hypervolumes covered after each trial and after combining all 15 trials are computed.

The per-trial median hypervolumes and 15-trials-combined hypervolumes obtained by HA1 are provided in Tables V and VI for all problem instances.

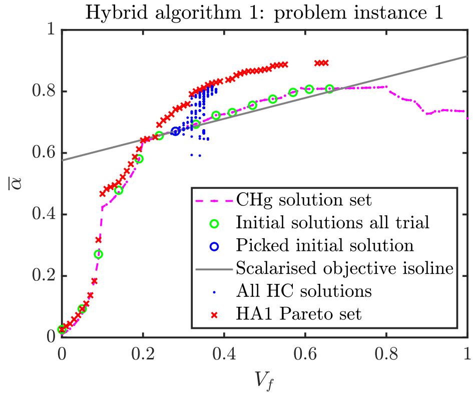


Figure 11. (color online) Hybrid approach 1 illustration of a trial for problem instance 1. Apply CHg for 25% of the budget. Pick an initial solution on the CHg Pareto set. Set scalarisation weight such that the isolines of the combined objective are tangential to the Pareto front at the selected CHg point. Apply hill climbing for the rest of the fitness evaluation budget. The final Pareto set after combining 15 trials each starting from equispaced points on the CHg Pareto set are shown using 'x' markers.

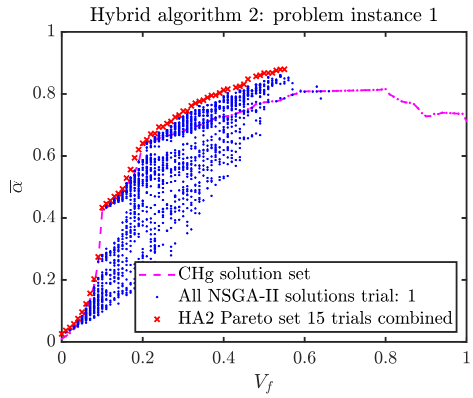


Figure 12. (color online) Hybrid approach 2: CHg run for 25% of computational budget, and then using the Pareto set as the initial population, NSGA-II is run for the remaining budget. Solutions traversed by NSGA-II in one of the 15 trials are shown in blue dots. The combined Pareto front from 15 trials is shown in red crosses.

B. Hybrid approach 2: CHg+NSGA-II

Hybrid approach 2 (HA2) combines CHg and NSGA-II in a similar fashion i.e., CHg uses 25% of the budget, NSGA-II uses the remaining 75%. The rationing of fitness evaluations is similar to that in HA1.

Originally, the final solution set from CHg was meant to be used as the initial population for NSGA-II in each trial. However, on some occasions the CHg Pareto front contained more or less solutions than the population size assigned for NSGA-II. Hence, when there were more solutions in CHg Pareto set, only 32 solutions equispaced in volume fraction were considered as the initial population for NSGA-II, and when there were less solutions, they were duplicated using the selection process in the first generation. Then NSGA-II is run for the remainder of the budget.

Figure 12 shows the solutions searched in an example trial out of the 15 trials that were run for problem instance 1. The combined Pareto front from 15 trials is then plotted using red crosses. It may be observed that in the low volume fraction regions, the solutions from NSGA-II never seem to improve. This is because crossover and mutation operations always produced worse solutions. The hypervolumes covered by the median trial and the overall hypervolume of the combined non-dominated solutions across 15 trials of HA2 are provided in Tables V and VI.

C. Overall comparison

1. Trial-averaged performance for 4096 budget

For a computational budget of 4096 gradient-free fitness evaluations, Table V shows the resulting hypervolumes covered by all algorithms used in this study. It should be noted that CHg did not need to use the entire budget. Since in each iteration, CHg has to fill at least one element, the entire design domain can be filled with only {100, 150, 1000, 100, 100, 1000, 50} fitness evaluations respectively for problem instances 1 through 7.

Keeping this in mind, the table shows that HA2, a combination of CHg and NSGA-II, covers the most hypervolume in 4 out of 7 problem instances on average per trial. Note that HA2 also performs better than stand-alone NSGA-II for the same budget. While it is evident that gradient-based initialisation boosts the performance of NSGA-II, it is interesting to note that HA2 can perform better that SIMPrestart or SIMPsweep which are normally used in practise. Thus, if one has a fixed computational budget, to cover the most hypervolume, a reliable strategy is to use a combination of CHg followed by NSGA-II.

Also, it is worth noting that SIMPsweep performs the best in two problem instances and CHg performs best in one problem instance. Notably, SIMPsweep and CHg are also scalable for lower budgets. These three algorithms may be recommended for applications such as software implementations in the initial stages of design that need to quickly come up with trade-off acoustic solutions within a set computational budget.

2. Combined performance of 15 trials each with 4096 budget

It is also of interest to identify effective strategies that find solutions with best attainable quality with relaxed computational time budgets, such as for manufacturing best acoustic designs. Table VI shows the resulting hypervolumes covered by a combination of 15 trials which is equivalent to 15*4096 gradient-free function evaluations. For this comparison, we do not include the gradient methods as they did not use the same budget.

In this study, HC shows a significant improvement as it is able to combine the good solutions from various regions of the Pareto front. For the same reason HA1 (CHg+HC) also performs exceptionally well, producing the best hypervolumes in 6 out of 7 problem instances. This shows that the proposed Pareto-slope-based weighted-sum scalarisation technique with a simple greedy hill climbing algorithm can be used as an effective local improvement strategy. A take-away is that before manufacturing an optimal shape using any multi-objective topology optimisation approach, it is worth ensuring that there exists no other dominating solution that HC can find.

Between NSGA-II and its hybrid counterpart HA2, the latter seems to cover more hypervolumes across all problem instances. This is again an example of a hybrid approach performing better than its parent approach. HA2 also performed the best in one of the seven problem instances, and comes close to the performance of HA1. This show that there is benefit to using hybrid strategies involving gradient initialisers with non-gradient improvers.

3. Pareto front comparison for all algorithms combined across 15 trials

The problem of topology optimisation has no exact algorithms that run in practical times to confirm the true Pareto-optimal solutions. Nevertheless, it is of interest to see which algorithms contribute to finding the best known solutions in the Pareto diagram.

Hence, we compare the Pareto fronts obtained from all algorithms in one place. As an example, this is shown for problem instance 1 in Figure 13. The gradient algorithms are marked in blue, non-gradient in red and hybrid in green.

It should be noted that the Pareto fronts for gradient algorithms are obtained from only one trial, while results for other algorithms are from a combination of 15 trials. Hence, one cannot draw a direct comparison across gradient strategies and others.

Among the three gradient algorithms, it may be observed that CHg finds better absorbing solutions in lower volume fractions up to 0.3, and the SIMP algorithms found better solutions after $V_f = 0.3$.

Among non-gradient algorithms, it is clear that all approaches perform better than random search, but there is no single clear winner between HC and NSGA-II.

Hybrid algorithms work best to cover the most hypervolume, but interestingly, there are some regions where HC produces better non-dominated solutions (see between V_f=0.1 and 0.3). This shows that one cannot ignore HC just because the hypervolume spanned is poor. The potential of HC for local exploration needs to be recognised.

Table V. Median hypervolumes obtained while running one trial with a budget equivalent to 4096 gradient-free fitness evaluations. HA2 seems to perform best when considering the trial-averaged performance for 4096 fitness evaluations.

	Gradient-based			Gradient-free			Hybrid	
Fitness evaluations	1365	1365	min(n_e/m,1365)	4096	4096	4096	4096	4096
Instance/ Alg	SIMPrestart	SIMPsweep	CHg	HC	NSGAII	RAND	HA1	HA2
1	0.7065	0.6835	0.6724	0.5622	0.6824	0.5915	0.7013	**0.7170**
2	0.4014	0.4047	0.4066	0.2684	0.3427	0.3212	0.4066	**0.4068**
3	0.7317	0.6063	**0.7412**	0.5908	0.6336	0.6061	0.7343	0.7184
4	0.1160	**0.1188**	0.1087	0.0893	0.1148	0.1085	0.1122	0.1174
5	0.5208	0.5292	0.5323	0.3798	0.4847	0.4561	0.5324	**0.5327**
6	0.7202	**0.7607**	0.7512	0.5430	0.6159	0.6211	0.7603	0.7601
7	0.8727	0.8567	0.8733	0.7133	0.8531	0.7677	0.8733	**0.8758**

Table VI. Hypervolume combined over 15 trials are compared in this table. These hypervolumes are also contrasted with those of single trials of gradient algorithms. HA1 seems to perform consistently better when considering a combination of 15 trials of 4096 fitness evaluations.

	Gradient-free			Hybrid	
Instance	HC	NSGAII	RAND	HA1	HA2
Budget	15*4096	15*4096	15*4096	15*4096	15*4096
1	0.7436	0.7302	0.6221	**0.7438**	0.7307
2	0.4029	0.3613	0.3329	**0.4081**	0.4074
3	0.7772	0.6878	0.6219	**0.8104**	0.7295
4	0.1144	0.1169	0.1107	**0.1195**	0.1190
5	0.5212	0.5034	0.4708	**0.5343**	0.5337
6	0.7509	0.6310	0.6269	**0.7646**	0.7606
7	0.8407	0.8725	0.8021	0.8755	**0.8759**

VI. CONCLUSION

In this article, several multi-objective strategies were compared to identify effective approaches for quickly obtaining lightweight and high-absorbing acoustic shape designs within a given amount of computational effort. Three gradient strategies—SIMPrestart, SIMPsweep and CHg, two gradient-free strategies—HC and NSGA-II, and two hybrid strategies—HA1 (CHg+HC) and HA2 (CHg+NSGA-II), were studied. The findings are highlighted as follows.

1. Gradient algorithms often get stuck at local-optimal shapes indicated by the fact that non-gradient approaches have been able to find better solutions in terms of both absorption and volume fraction objectives.
2. Reusing solutions from SIMP with an adaptive volume fraction constraint (SIMPsweep) is better at spanning the Pareto front than restarting SIMP at various volume fraction constraints (SIMPrestart).
3. A simple new gradient-based constructive heuristic (CHg) is introduced that guarantees discrete solutions while also being scalable and as performant as SIMP algorithms.
4. Hybrid approaches using gradient algorithms as initialisers and non-gradient algorithms as exploiters seem to be more effective than any parent gradient or non-gradient algorithm for the same computational budget.
5. Hill climbing with a Pareto-slope-based weighted-sum scalarisation proves to be an effective local search technique to improve solutions near the Pareto front.

If the goal is to quickly find a set of trade-off shapes, such as to use in software applications, then any gradient approach or a hybrid approach with CHg and NSGA-II would be more suitable. If

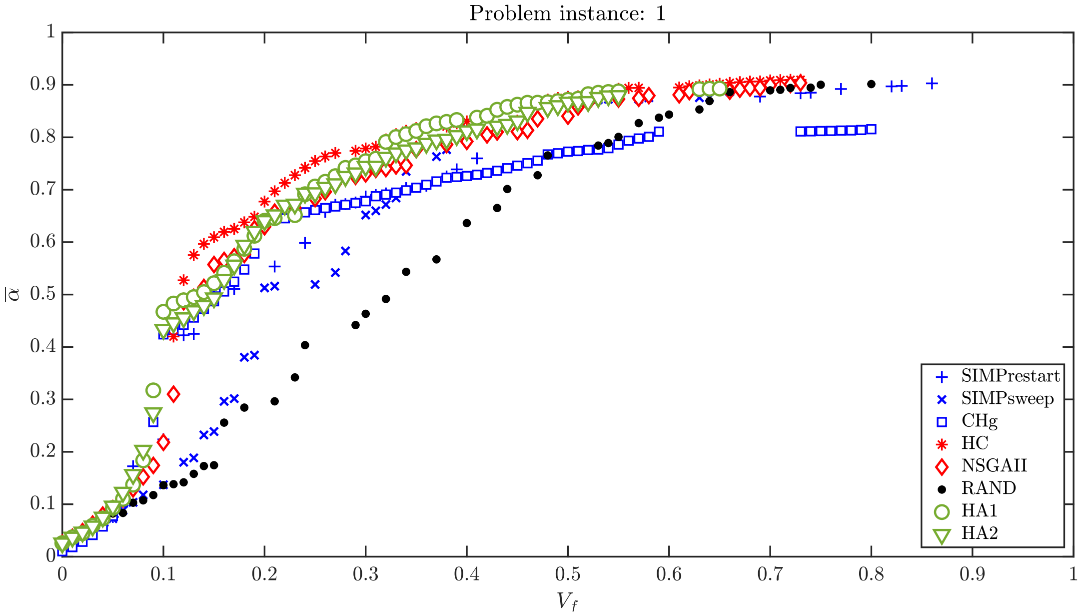


Figure 13. (color online) Comparison of non-dominated solutions from all algorithms for problem instance 1. Colours blue, red and green correspond to gradient, non-gradient and hybrid algorithms respectively. Gradient algorithm results are from one trial, whereas non-gradient and hybrid algorithm results are from a combination of 15 trials. Hence they must not be compared.

the goal is to obtain the optimised shape designs of the best attainable quality for manufacture, then a hybrid approach with CHg and hill climbing with a Pareto-slope-based scalarisation seems to be more suitable. If the interest is to find the best attainable trade-off solutions to a problem, then no algorithm is a clear winner. Algorithms such as HC occasionally find better solutions in specific regions than their hybrid counterpart and cannot be ignored.

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